Homework 1

Due date: Jan 30, 2020, 11:55PM.

Bibliography: Trefethen & Bau, Lectures 1-8. Problems 1-4 = 1 pt each, Problem 5 = 4 points.

1. Exercises 2.3, 2.4, 2.5

2. Exercises 3.1-3.6

3. Exercise 5.4. State and solve the analogous problem for skew-symmetric matrices.

4. Exercises 6.1-3, 6.5

5. Consider a black and white image represented as a matrix $A \in \{0, 1\}^{32 \times 256}$. In each 32×32 block set element values to represent a letter of the words: "absolute", "computer", "measures".

   a) Guess the rank of the matrices. Then, compute the rank of the matrices.

   b) Obtain a sequence of approximations A_ν for $\nu = 2^p$, $p = 2, 3, 4, 5$, with A_ν the successive approximations from truncation of the SVD rank-1 expansions.

   c) Consider $B(x_1, x_2, x_3) = x_1 A_1 + x_2 A_2 + x_3 A_3$, with $x_1 + x_2 + x_3 = 1$. Repeat (b) for x_1, x_2, x_3 by sampling points within the tetrahedron. Comment.

   d) Consider $H(\xi) = A_1 + \xi(A_2 - A_1)$. Repeat (b) for $\xi \in (0, 1)$. Comment.