Homework 4&5&6

Due date: May 5, 2021, 11:55PM.
Bibliography: Trefethen & Bau, Lectures 24-39

1. Use the Schur decomposition to prove:

   Theorem. *Abel's Formula. If* $\boldsymbol{A}(t)$ *is continuous on* (a,b)*, and* $\boldsymbol{Y} = (\ \boldsymbol{y}_1\ \ \ldots\ \ \boldsymbol{y}_n\ )$ *is a matrix whose columns are solutions of* $\boldsymbol{y}' = \boldsymbol{A}\boldsymbol{y}$ *on* (a,b)*, then the Wronskian* $W(t) = |\boldsymbol{Y}|$ *is given by*

$$W(t) = \exp\left[\int_{t_0}^{t} \operatorname{tr} \boldsymbol{A}(s)\,\mathrm{d}s\right] W(t_0).$$

2. Ex. 27.3, 27.5, pp. 209-210.
3. Ex 29.1, pp. 223-224
4. Ex 30.4, p. 233
5. Ex. 36.2, p. 283.
6. Ex. 36.3, p. 284.
7. Ex. 36.4, p. 284 (modify Lesson20.ipynb to plot eigenvalues and Ritz estimates, and lemniscates also).
8. Ex. 38.5, p. 302
9. Ex. 38.6, p. 302 (the code framework for this problem will be developed in class).
10. Ex. 39.5 p. 312

11. Apply the code from Ex.3 to find elastic vibrational modes (eigenvalues and eigenvectors) of the L-shaped plate shown in the figure below. The system matrix is generated from the FreeFEM code derived in class.
12. Also find the elastic vibrational modes through Lanczos iteration, plotting intermediate Lanczos lemniscates. Compare with results from Ex. 11.