Overview

- Givens rotators
- Least squares:
  - orthogonal projection
  - normal equations
  - pseudo-inverse

- In 2D

Reflector

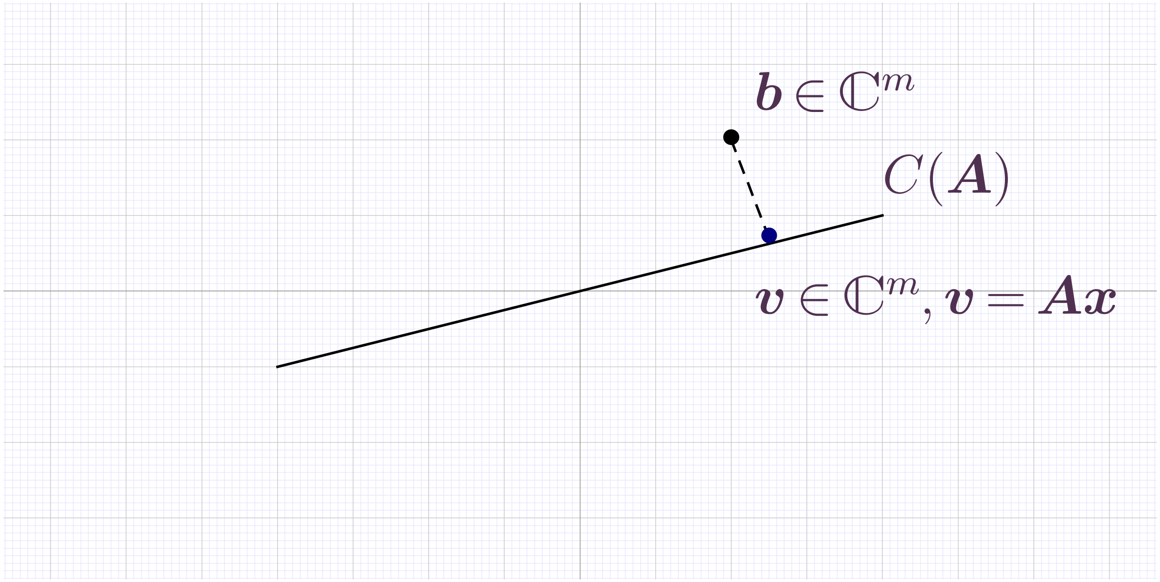
$$F = \begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$F \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1\cos\theta + x_2\sin\theta \\ x_1\sin\theta + x_2\cos\theta \end{bmatrix}$$

Rotator

$$J = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$J \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1\cos\theta + x_2\sin\theta \\ -x_1\sin\theta + x_2\cos\theta \end{bmatrix}$$



$$\min_{\boldsymbol{x} \in \mathbb{C}^n} \|\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}\|, \boldsymbol{x} = \arg\min_{\boldsymbol{y} \in \mathbb{C}^n} \|\boldsymbol{b} - \boldsymbol{A}\boldsymbol{y}\|$$

$$\boldsymbol{A} = \boldsymbol{Q}\boldsymbol{R}, \boldsymbol{v} = \boldsymbol{P}_{C(\boldsymbol{A})}\boldsymbol{b} = \boldsymbol{P}_{\boldsymbol{Q}}\boldsymbol{b} = \boldsymbol{Q}\boldsymbol{Q}^*\boldsymbol{b} = (\boldsymbol{Q}\boldsymbol{R})\boldsymbol{x} \Rightarrow \boldsymbol{R}\boldsymbol{x} = \boldsymbol{Q}^*\boldsymbol{b}$$

$$\mathcal{D} = \{(x_i, y_i), i = 1, \ldots, m\}$$

$$p_{m-1}(x) = c_0 + c_1 x + \cdots + c_{m-1} x^{m-1} = \begin{bmatrix} 1 & x & \ldots & x^{m-1} \end{bmatrix} \boldsymbol{c}$$

$$\min \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{c}\|$$

```
octave> m=1000; h=1./m; x=(1:m)'*h; y=1-2*x+3*x.^2-4*x.^3;
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```
octave> A=[x.^0 x.^1 x.^2 x.^3]; [Q,R]=qr(A);
```

```
octave> c=R\Q'*y
```

```
octave> c
```

$$\begin{pmatrix} 1 \\ -2 \\ 3 \\ -4 \end{pmatrix}$$

```
octave>
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