Overview

# Log Least squares, normal equations, pseudo-inverse 

Overview



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MATH662: L09 Least squares, normal equations, pseudo-inverse D
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#### Abstract

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Theorem. Given $\boldsymbol{A} \in \mathbb{C}^{m \times n}, \boldsymbol{b} \in \mathbb{C}^{m}, m \geqslant n, x \in \mathbb{C}^{n}$ minimizes the norm of the residual $\boldsymbol{r}=\boldsymbol{b}-\boldsymbol{A} \boldsymbol{x}$ iff $\boldsymbol{r} \perp C(\boldsymbol{A})$, e.g.

$$
\boldsymbol{A}^{*} \boldsymbol{r}=0 \Leftrightarrow \boldsymbol{A}^{*} \boldsymbol{A} \boldsymbol{x}=\boldsymbol{A}^{*} \boldsymbol{b} \Leftrightarrow \boldsymbol{P b}=\boldsymbol{A} \boldsymbol{x}
$$

with $\boldsymbol{P}$ the orthogonal projector onto $C(\boldsymbol{A})$. The normal equation system

$$
\boldsymbol{A}^{*} \boldsymbol{A} \boldsymbol{x}=\boldsymbol{A}^{*} \boldsymbol{b}
$$

has $\boldsymbol{M}=\boldsymbol{A}^{*} \boldsymbol{A}$ nonsingular iff $\operatorname{rank}(\boldsymbol{A})=n$.

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－If \(\operatorname{rank}(\boldsymbol{A})=n\) ，then \(\boldsymbol{x}=\left(\boldsymbol{A}^{*} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^{*} \boldsymbol{b}=\boldsymbol{A}^{+} \boldsymbol{b}\) is the solution of the least squares problem， －If \(\operatorname{rank}(\boldsymbol{A})=n\) ，then \(\boldsymbol{x}=\left(\boldsymbol{A}^{*} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^{*} \boldsymbol{b}=\boldsymbol{A}^{+} \boldsymbol{b}\) is
with \(\boldsymbol{A}^{+}=\left(\boldsymbol{A}^{*} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^{*}\) the pseudo－inverse of \(\boldsymbol{A}\)
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#### Abstract




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