



1 2 3 4 5 6

- Through QR -factorization system $\mathbf{Ax} = \mathbf{b}$ is reduced to $\mathbf{Rx} = \mathbf{y}$, with $\mathbf{R} \in \mathbb{C}^{m \times m}$ upper triangular. The solution is found by backward substitution

for $i = m$ downto 1

$$x_i = y_i / r_{ii}$$

for $j = 1$ to $i - 1$

$$y_j = y_j - r_{ij} x_i$$

- Backward substitution is backward stable

Theorem. For $\mathbf{R} \in \mathbb{C}^{m \times m}$ upper triangular, $\mathbf{y} \in \mathbb{C}^m$, there exists $\delta\mathbf{R}$ with $\|\delta\mathbf{R}\| / \|\mathbf{R}\| = \mathcal{O}(\epsilon_{\text{mach}})$ such that the approximate result $\tilde{\mathbf{x}}$ given by the backward substitution algorithm $(\mathbf{R}, \mathbf{y}) \rightarrow^{\tilde{\mathbf{f}}} \tilde{\mathbf{x}}$ satisfies $(\mathbf{R} + \delta\mathbf{R})\tilde{\mathbf{x}} = \mathbf{y}$.

- Since Householder QR -factorization and backward substitution are both backward stable, solving $\mathbf{Ax} = \mathbf{b}$ through these two methods is also backward stable