Test 1

Solve the following problems (4 course points each). Present a brief motivation of your method of solution.

1. Let $\boldsymbol{A}^+(x)$ denote the pseudoinverse of $\boldsymbol{A}(x) \in \mathbb{R}^{2\times 2}$ defined as

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 \\ 0 & x \end{bmatrix}, x \in \mathbb{R}.$$

   a) Is $\boldsymbol{A}^+(x)$ continuous at $x = 0$?

   b) Estimate the error $e = \|\boldsymbol{A}^+(\varepsilon) - \boldsymbol{A}^+(0)\|_2$ for small ε.

   c) Comment on floating-point computational implications of your results from (a) and (b).

2. Find the singular value decomposition of

$$\boldsymbol{A} = \begin{bmatrix} -4 & -6 \\ 3 & -8 \end{bmatrix},$$

   showing all intermediate steps.

3. Let $\boldsymbol{A}^+$ denote the pseudoinverse of $\boldsymbol{A} \in \mathbb{C}^{m\times n}$. Express the four fundamental vector subspaces of $\boldsymbol{A}^+$ in terms of those of $\boldsymbol{A}$.