MATH761: Numerical ODE/PDE I

Course syllabus

Times	MWF 10:10-11:00AM, Phillips 224
Office hours	Th 12:30-1:30PM, and by email appointment, Chapman 451
Instructor	Sorin Mitran

This graduate course presents the theory and application of numerical approaches to solution of differential equations. Both ordinary differential equations (ODEs) and partial differential equations (PDEs) are discussed.

The instructor reserves the right to make changes to the syllabus. Any changes will be announced as early as possible.

Course goals

Students will acquire proficiency in the formulation of numerical schemes for solving ODEs and PDEs using finite difference, finite volume, finite element, boundary element, and spectral methods. A broad overview of each approach will be discussed. Depending on class interest, specific methods will be studied in more detail. Application are chosen from domains such as fluid dynamics, rheology, elasticity, plasticity.

Honor Code

Unless explicitly stated otherwise, all work is individual. You may discuss various approaches to homework problems with students, instructors, but must draft your answers by yourself. In joint projects, each student will clearly identify which portions of the work they contributed.

Grading

Required work

- Homework, 6 assignments x 12 = 72 points
- Midterm examination = 12 points
- Final project = 16 points

Mapping of point scores to letter grades

Grade	**Points**	**Grade**	**Points**	**Grade**	**Points**	**Grade**	**Points**
H+,A *cum laude*	101-118	H-,B+	86-90	P-,C+	71-75	L-,D+	56-60
H+,A	96-100	P+,B	81-85	L+,C	66-70	L–,D-	50-55
H,A-	91-95	P,B-	76-80	L,C-	61-65	F	0-49

Course policies

- This is a graduate-level course. Students are expected to undertake independent work to deepen their understanding of course material
- Students are free to establish their own schedule; there is no need to inform instructor of absences. Course attendance is highly recommended to gain insight into course topics
- Late homework is not accepted.

- Homework is to be submitted in electronic form through Sakai, as a paper for publication in SIAM typeset format. Templates will be provided.
- Computer code should be submitted in commented, correct, ready-to-run form, and directly reflect theory and conclusions from the homework submission. Code is to be submitted in the form of a supplement to a SIAM paper
- Homework assignments are inspired by the real-world practice of scientific computation.

Examinations

- A midterm examination will given on topics within ODE and finite difference theory
- The final examination is a project going into greater detail on a subject chosen from the homeworks

Course materials

Course topics

- Ordinary differential equations (ODEs): Convergence, consistency, stability; Single-step, multi-step methods; Boundary value problems
- Finite difference methods and analysis (FDM): Convergence, consistency, stability; Modified equations; Fourier analysis of finite difference methods
- Finite volume methods (FVM): Godunov methods and Riemann problems, Essentially non-oscillatory schemes, Central schemes, Adaptive mesh refinement
- Spectral methods (SM): Operator eigenfunction expansions; Collocation methods; Riesz theorem; Convergence analysis
- Finite element methods (FEM): Grid generation; Finite element spaces; System matrix assembly; Variational formulations
- Boundary element methods (BEM): Convolution solutions, fast solvers
- Adaptive mesh refinement (AMR)
- Multigrid methods (MG)

Bibliography

There is no single course text. Topics are drawn from the following sources

- *Course lecture notes*, S. Mitran
- *Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems*, R. LeVeque
- *Finite Volume Methods for Hyperbolic Problems* by R. LeVeque,
- *Spectral Methods in Fluid Dynamics*, by C. Canuto, Y. Hussaini, A. Quarteroni, T. Zhang,
- *The Finite Element Method: Its Basis and Fundamentals*, by O. Zienkiewicz, R. Taylor, J. Zhu
- *Discontinuous Galerkin Methods*, by B. Cockburn, G. Karniadakis, C.-W. Shu

Class notes

Notes are posted 48 hours prior to class time, and are generally specified as sections from *Course lecture notes*. Additional notes are posted as needed. Read notes before class to gain a first exposure to lecture material.

Week	Dates	Monday	Wednesday	Friday
01	08/20-24	-	Lesson01: ODEs §3.1-3	Lab01: Intro
02	08/27-31	Lesson02: ODEs	Lesson03: ODEs	Lab02: ODEs
03	09/03-07	(Labor Day)	Lesson04: BVP	Lab03: BVP
04	09/10-14	Lesson06: FDM	Lesson07: FDM	Lab04: FDM
05	09/17-21	Lesson08: FDM	Lesson10: FDM	Lab05: FDM
06	09/24-28	Lesson11: FVM	Lesson12: FVM	Lab06: FVM
07	10/01-05	Lesson13: FVM	Lesson14: FVM	Lab07: FVM
08	10/08-12	Lesson15: SM	Lesson16: SM	(University Day)
09	10/15-19	Lesson17: SM	Midterm exam	(Fall Break)
10	10/22-26	Lesson18: SM	Lesson19: SM	Lab08: SM
11	10/29-02	Lesson20: FEM	Lesson21: FEM	Lab09: FEM
12	11/05-09	Lesson22: FEM	Lesson23: BEM	Lab10: BEM
13	11/12-16	Lesson24: BEM	Lesson25: AMR	Lab11: AMR
14	11/19-23	Lesson26: MG	(Thanksgiving)	(Thanksgiving)
15	11/26-30	Lesson27: MG	Lesson28: MG	Lab12: MG
16	12/03-07	Review	Review	-

Homework

Homework assignments continue the computational labwork.

Nr.	Issue Date	Due Date	Topic	Problems	Solutions
1	08/29	09/12	ODEs & BVP		
2	09/12	09/26	FDM		
3	09/26	10/10	FVM		
4	10/10	10/24	SM		
5	10/24	11/07	FEM & BEM		
6	11/07	11/26	AMR & MG		
Project	11/26	12/05			

Computational lab

SciComp@UNC Linux environment

Scientific computation is typically carried out in a Un*x environment (e.g. OS/X, various Linux versions). This course uses a customized Linux environment named SciComp@UNC available to students as a virtual machine. Download Virtual Box and the SciComp@UNC virtual machine image.

Various open source tools for carrying out and documenting practical scientific computation will be successively introduced:

- TeXmacs: editing of documents containing live computation
- SciPy: scientific Python environment
- Maxima: symbolic computation
- Octave: matrix operations through a Matlab clone
- Gnu compilers (Fortran,C,Go): high-performance code development
- Julia: a high-performance interactive environment
- Paraview: data visualization
- OpenDX: data visualization with a graphical editor
- Gnuplot: simple data plotter
- BEARCLAW: a package for solving partial differential equations

The course will also use a few commercial tools, freely available to students while connected to the campus network (either directly or remotely through the UNC VPN server):

- Mathematica: complete system for numerical, symbolic, and graphical computation
- Totalview: an interactive debugger for compiled code

Course labwork

The course will focus on application of theoretical concepts to realistic examples. Academic examples are introduced in the weekly lab, and extended to realistic applications in the bi-weekly homework assignments.

Course material repository

Course materials (lecture notes, workbooks, homework, examination examples) are stored in a repository that is accessed through the subversion utility, available on all major operating systems. The URL of the material is http://mitran-lab.amath.unc.edu/courses/MATH761.

The above address is needed for an initial checkout using commands such as:

```
mkdir ~/courses
cd ~/courses
svn co svn://mitran-lab.amath/unc.edu/courses/MATH761/
```

In the SciComp@UNC virtual machine the initial checkout can be carried out through the terminal commands

```
cd ~/courses
make MATH761
```

Update the course materials before each lecture by:

```
cd ~/courses
svn update
```

Links to course materials will also be posted to this site, but the most up-to-date version is that from the subversion repository, so carry out the svn update procedure prior to each lecture.