LAB07: FINITE VOLUME METHOD EXAMPLES

1 Conservation laws

Consider the conservation law

$$\boldsymbol{q}_t + \boldsymbol{f}(\boldsymbol{q})_x + \boldsymbol{g}(\boldsymbol{q})_y = 0.$$

and the Riemann problem

$$\boldsymbol{q}(0,x,y) = \begin{cases} \boldsymbol{q}_l & x<0 \\ \boldsymbol{q}_r & x>0 \end{cases} \tag{1}$$

for $t>0, -4\pi \leqslant x \leqslant 4\pi$.

The above problem is solved in BEARCLAW for:

1. Advection equation $q_t + (uq)_x + (vq)_y = 0$
2. Burgers equation $q_t + \left(\frac{1}{2}q^2\right)_x = 0$
3. Wave equation $\varphi_{tt} - c^2\nabla^2\varphi = 0$, transformed to system $\boldsymbol{q}_t + \boldsymbol{A}\boldsymbol{q}_x + \boldsymbol{B}\boldsymbol{q}_y = \boldsymbol{0}$, with notation $(u,r,s) = (\varphi_t, \varphi_x, \varphi_y)$

$$\boldsymbol{q} = \begin{pmatrix} u \\ r \\ s \end{pmatrix}, \boldsymbol{A} = -\begin{pmatrix} 0 & c^2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \boldsymbol{B} = -\begin{pmatrix} 0 & 0 & c^2 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

4. Euler equations of gas dynamics

$$\boldsymbol{q} = \begin{pmatrix} \rho \\ l \\ m \\ \varepsilon \end{pmatrix}, \boldsymbol{f}(\boldsymbol{q}) = \begin{pmatrix} l \\ \frac{l^2}{\rho} + p \\ \frac{lm}{\rho} \\ lH \end{pmatrix}, \boldsymbol{g}(\boldsymbol{q}) = \begin{pmatrix} m \\ \frac{lm}{\rho} \\ \frac{m^2}{\rho} + p \\ mH \end{pmatrix},$$

with $l = \rho u$, $m = \rho v$, $\varepsilon = \rho E$, $H = E + \frac{1}{2}(u^2+v^2)$, $p = \rho R T = (\gamma-1)\left(\varepsilon - \frac{l^2+m^2}{\rho}\right)$.

Each of the above problems will be solved in BEARCLAW using literate programming techniques. See lab07 subdirectories for each problem.