Problem. Study the accuracy and stability of finite difference discretizations of the IBVP for the heat equation

$$q_t = q_{xx} \text{ in } (0,1), q(t=0,x) = f(x), q(t,0) = 0, q(t,1) = 0$$

Approach:

- Obtain analytical solution
- Use finite difference calculus to define approximations of $\mathcal{O}(k^r, h^p)$ of differentiation operations
- Generate Fortran code implementations to carry out numerical experiments
- Analyze stability

Solution. By separation of variables:

$$q(t,x) = \sum_{n=1}^{\infty} B_n \sin(n\pi x)\, e^{-(n\pi)^2 t} \cong \sum_{n=1}^{P} B_n \sin(n\pi x)\, e^{-(n\pi)^2 t}, B_n = \langle f, b_n \rangle = 2\int_0^1 f(x) \sin(n\pi x)\, \mathrm{d}x.$$

Example. $f(x) = \sin(\pi x), P = 4, \boldsymbol{B} = \{1,0,0,0\}$

$$q(t,x) = e^{-\pi^2 t} \sin(\pi x)$$

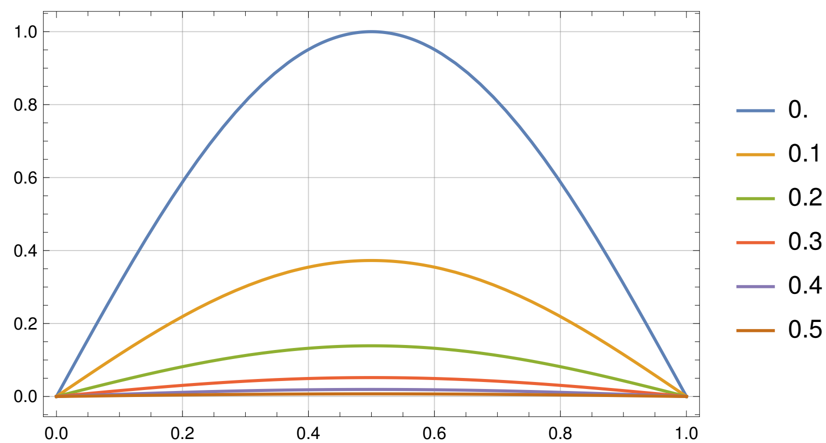

Figure 1. Heat IBVP solution at $t = \{0., 0.1, 0.2, 0.3, 0.4, 0.5\}$.