Definition. *For grid data* $\mathbf{Q}_i = \{Q_{i+j} \cong q(x_i + jh), j \in J\}$, *a finite difference formula* $\mathcal{F}_h^k(\mathbf{Q}_i)$ *is a* p^{th}*-order accurate approximation of the* k^{th} *derivative* $q^{(k)}(x_i)$ *if* $\|\mathcal{F}_h^k(\mathbf{Q}_i) - q^{(k)}(x_i)\| = \mathcal{O}(h^p)$.

⇑ Accuracy analysis in the independent variable space is carried out by Taylor series comparison

```
In[9]:= Dp[f[x],1]
```

$$\frac{f(h+x) - f(x)}{h}$$

```
In[11]:= Series[ Dp[f[x],1] ,{h,0,2}] - f'[x]
```

$$\frac{1}{2} h f''(x) + \frac{1}{6} h^2 f^{(3)}(x) + O(h^3)$$

```
In[24]:= Table[ {(Dp[f[i],p] /. f->Q /. h->1)/h,Limit[(Normal[Series[Dp[f[x],p],{h,0,p}]] - f'[x])/h^p,
         h->0] /. f->q},{p,1,5}] // TableForm
```

$\frac{Q_{i+1} - Q_i}{h}$	$\frac{q''(x)}{2}$
$\frac{-3 Q_i + 4 Q_{i+1} - Q_{i+2}}{2h}$	$-\frac{1}{3} q^{(3)}(x)$
$\frac{-11 Q_i + 18 Q_{i+1} - 9 Q_{i+2} + 2 Q_{i+3}}{6h}$	$\frac{1}{4} q^{(4)}(x)$
$\frac{-25 Q_i + 48 Q_{i+1} - 36 Q_{i+2} + 16 Q_{i+3} - 3 Q_{i+4}}{12h}$	$-\frac{1}{5} q^{(5)}(x)$
$\frac{-137 Q_i + 300 Q_{i+1} - 300 Q_{i+2} + 200 Q_{i+3} - 75 Q_{i+4} + 12 Q_{i+5}}{60h}$	$\frac{1}{6} q^{(6)}(x)$

```
In[25]:=
```

- Symmetry of Taylor series expansion of centered schemes leads to higher-order

⇑ Accuracy analysis in the independent variable space is carried out by Taylor series comparison

```
In[25]:= d[f[x],1]
```

$$\frac{f\left(\frac{h}{2}+x\right)-f\left(x-\frac{h}{2}\right)}{h}$$

```
In[26]:= Series[ d[f[x],1] ,{h,0,2}] - f'[x]
```

$$\frac{1}{24}h^2 f^{(3)}(x)+O(h^3)$$

```
In[12]:= Table[ {(d[f[i],p] /. f->Q /. h->1)/h,Limit[(Normal[Series[d[f[x],p],{h,0,2p}]] - f'[x])/
         h^(2p), h->0] h^(2p) /. f->q},{p,1,3}] // TableForm
```

$\frac{Q_{i+\frac{1}{2}}-Q_{i-\frac{1}{2}}}{h}$	$\frac{1}{24}h^2 q^{(3)}(x)$
$\frac{Q_{i-\frac{3}{2}}-27\,Q_{i-\frac{1}{2}}+27\,Q_{i+\frac{1}{2}}-Q_{i+\frac{3}{2}}}{24\,h}$	$-\frac{3}{640}h^4 q^{(5)}(x)$
$\frac{-9\,Q_{i-\frac{5}{2}}+125\,Q_{i-\frac{3}{2}}-2250\,Q_{i-\frac{1}{2}}+2250\,Q_{i+\frac{1}{2}}-125\,Q_{i+\frac{3}{2}}+9\,Q_{i+\frac{5}{2}}}{1920\,h}$	$\frac{5\,h^6 q^{(7)}(x)}{7168}$

```
In[13]:=
```

- Taylor series expansion of formulas from repeated application of finite difference operators

⇑ Accuracy analysis in the independent variable space is carried out by Taylor series comparison

```
In[16]:= Dp[f[x],1,2]
```

$$\frac{-2\, f\,(h+x)+f\,(2\,h+x)+f(x)}{h^2}$$

```
In[17]:= Series[ Dp[f[x],1,2] ,{h,0,2}] - f''[x]
```

$$h\, f^{(3)}(x)+\frac{7}{12}h^2\, f^{(4)}(x)+O(h^3)$$

```
In[21]:= Table[ {(Dp[f[i],p,2] /. f->Q /. h->1)/h,Limit[(Normal[Series[Dp[f[x],p,2],{h,0,p}]] - f'[x])/
         h^(p-1), h->0] /. f->q},{p,1,3}] // TableForm
```

$\frac{Q_i-2\,Q_{i+1}+Q_{i+2}}{h}$	$q''(x)-q'(x)$
$\frac{9\,Q_i-24\,Q_{i+1}+22\,Q_{i+2}-8\,Q_{i+3}+Q_{i+4}}{4\,h}$	Indeterminate
$\frac{121\,Q_i-396\,Q_{i+1}+522\,Q_{i+2}-368\,Q_{i+3}+153\,Q_{i+4}-36\,Q_{i+5}+4\,Q_{i+6}}{36\,h}$	$\infty\,(2\,q''(x)-2\,q'(x))$

```
In[22]:=
```

- Taylor series expansion of formulas from repeated application of finite difference operators

⇑ Accuracy analysis in the independent variable space is carried out by Taylor series comparison

```
In[22]:= d[f[x],1,2]
```

$$\frac{f(x-h)+f(h+x)-2f(x)}{h^2}$$

```
In[23]:= Series[ d[f[x],1,2] ,{h,0,2}] - f''[x]
```

$$\frac{1}{12}h^2 f^{(4)}(x)+O(h^3)$$

```
In[25]:= Table[ {(Dp[f[i],p,2] /. f->Q /. h->1)/h,Limit[(Normal[Series[Dp[f[x],p,2],{h,0,p}]] - f'[x])/
         h^(2p), h->0] /. f->q},{p,1,3}] // TableForm
```

$\frac{Q_i-2Q_{i+1}+Q_{i+2}}{h}$	$\infty(q''(x)-q'(x))$
$\frac{9Q_i-24Q_{i+1}+22Q_{i+2}-8Q_{i+3}+Q_{i+4}}{4h}$	$\infty(3q''(x)-3q'(x))$
$\frac{121Q_i-396Q_{i+1}+522Q_{i+2}-368Q_{i+3}+153Q_{i+4}-36Q_{i+5}+4Q_{i+6}}{36h}$	$\infty(2q''(x)-2q'(x))$

```
In[26]:=
```