Definition. *For grid data* $\mathbf{Q}_i = \{Q_{i+j} \cong q(x_i + jh), j \in J\}$, *a finite difference formula* $\mathcal{F}_h^k(\mathbf{Q}_i)$ *is a* p^{th}*-order accurate approximation of the* k^{th} *derivative* $q^{(k)}(x_i)$ *if* $\|\mathcal{F}_h^k(\mathbf{Q}_i) - q^{(k)}(x_i)\| = \mathcal{O}(h^p)$.

⇑ Accuracy analysis in the independent variable space is carried out by Taylor series comparison

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In[9]:= Dp[f[x],1]
```

$$\frac{f(h+x) - f(x)}{h}$$

```
In[11]:= Series[ Dp[f[x],1] ,{h,0,2}] - f'[x]
```

$$\frac{1}{2} h f''(x) + \frac{1}{6} h^2 f^{(3)}(x) + O(h^3)$$

```
In[24]:= Table[ {(Dp[f[i],p] /. f->Q /. h->1)/h,Limit[(Normal[Series[Dp[f[x],p],{h,0,p}]] - f'[x])/h^p,
         h->0] /. f->q},{p,1,5}] // TableForm
```

$\frac{Q_{i+1} - Q_i}{h}$	$\frac{q''(x)}{2}$
$\frac{-3 Q_i + 4 Q_{i+1} - Q_{i+2}}{2h}$	$-\frac{1}{3} q^{(3)}(x)$
$\frac{-11 Q_i + 18 Q_{i+1} - 9 Q_{i+2} + 2 Q_{i+3}}{6h}$	$\frac{1}{4} q^{(4)}(x)$
$\frac{-25 Q_i + 48 Q_{i+1} - 36 Q_{i+2} + 16 Q_{i+3} - 3 Q_{i+4}}{12h}$	$-\frac{1}{5} q^{(5)}(x)$
$\frac{-137 Q_i + 300 Q_{i+1} - 300 Q_{i+2} + 200 Q_{i+3} - 75 Q_{i+4} + 12 Q_{i+5}}{60h}$	$\frac{1}{6} q^{(6)}(x)$

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In[25]:=
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