

MATH762 Homework 2 - Finite element methods

Issued 2/21/12, Due 3/1/12

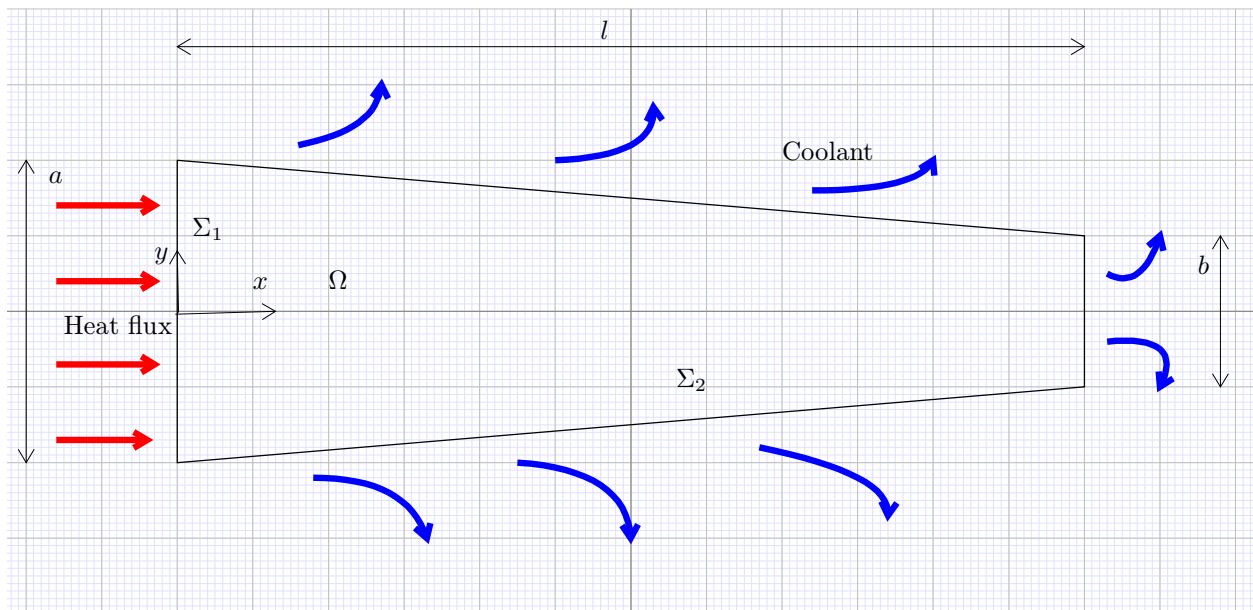
FreeFEM software

1. Download and install FreeFEM++-cs from
<http://www.ann.jussieu.fr/~lehyaric/ffcs/install.php>
2. Go through the tutorial
<http://www.ann.jussieu.fr/~lehyaric/ffcs/learning.php>
3. Browse through the documentation
<http://www.ann.jussieu.fr/~lehyaric/ffcs/doc/ff/ffdoc.html>
4. Study the thermal conduction problem at
<http://www.ann.jussieu.fr/~lehyaric/ffcs/doc/ff/ffdocch3.html#x7-350003.4>

Heat conduction in a cooling fin

I. Problem description

In order to rapidly cool a region fin shapes are often placed on the periphery to efficiently conduct heat away towards circulating coolant flow. Examples: internal combustion engines, refrigeration coils, etc.



The mathematical statement of the problem is

$$q_t = \nabla \cdot (\alpha \nabla q) \text{ in } \Omega$$

$$\frac{\partial q}{\partial n} = Q \text{ on } \Sigma_1$$

$$q = T \text{ on } \Sigma_2$$

$$q(x, y, t = 0) = T$$

1. Choose a, b, l, Q, T . Set $\alpha = \text{constant}$. Solve the problem and present contour lines of $q(x, y, t)$ at $t = 1, 2, \dots, 10$. Discretize the domain with $n = 50, 100, 150, 200, 250$ points along the boundary. Compare solutions. Start with elements resulting from $n = 50$ and carry out grid adaptation. Compare with grid from $n = 250$.

2. Consider $\alpha = 1 + x$ and repeat above computation.
3. Consider $\alpha = 1 + q$ and solve the resulting nonlinear problem by iteration. Use grid adaptation and start from the $n = 200$ domain discretization.