

MATH 762 - Homework 1

Due: 2/14/13

1. Consider the shallow water system

$$q_t + f(q)_x = 0$$

$$q = \begin{pmatrix} h \\ hu \end{pmatrix}, f(q) = \begin{pmatrix} hu \\ hu^2 + \frac{gh^2}{2} \end{pmatrix}$$

with units chosen such that $g = 1$, and the dam-break problem

$$q_l = q(x < 0, t = 0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, q_r = q(x > 0, t = 0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

- a) Derive the formulas for the Hugoniot loci and integral curves of the system
 - b) Draw the Hugoniot loci and integral curves that pass through q_l and q_r using some software package (e.g. Mathematica)
 - c) Find the solution to the Riemann problem. First estimate graphically from your plots within 1.b. Then solve algebraically.
2. Implement the shallow water system within BEARCLAW and carry out a convergence study of the numerical solution to the analytical solution from 1.
3. Apply the Lax-Wendroff scheme and compare with the BEARCLAW solution.