# Qualitative Fourier solution of PDEs

Wednesday, January 25, 2017 11:32 AM

(1) $\rho_t = -u\rho_x$

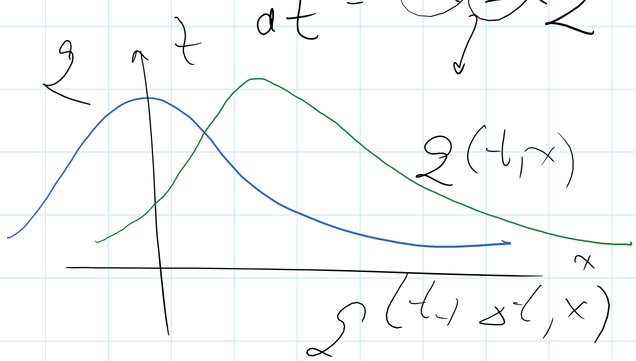
(2) $\rho_t = a\rho_{xx}$

(3) $\rho_t = \nu\rho_{xxx}$

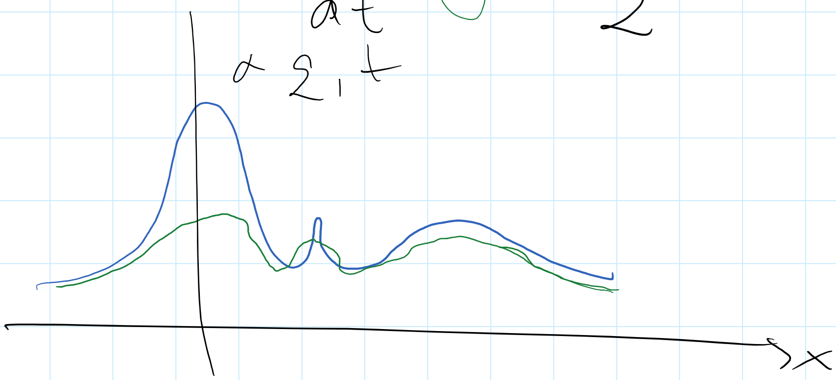
$$\rho(t,x)$$

$$\hat{\rho}(t,\xi) = \mathcal{F}\rho = \int_{-\infty}^{\infty} \rho(t,x)\, e^{-i\xi x}\, dx$$

$\mathcal{F}$ (1): $\dfrac{d\hat{\rho}}{dt} = -i\xi u\,\hat{\rho}$

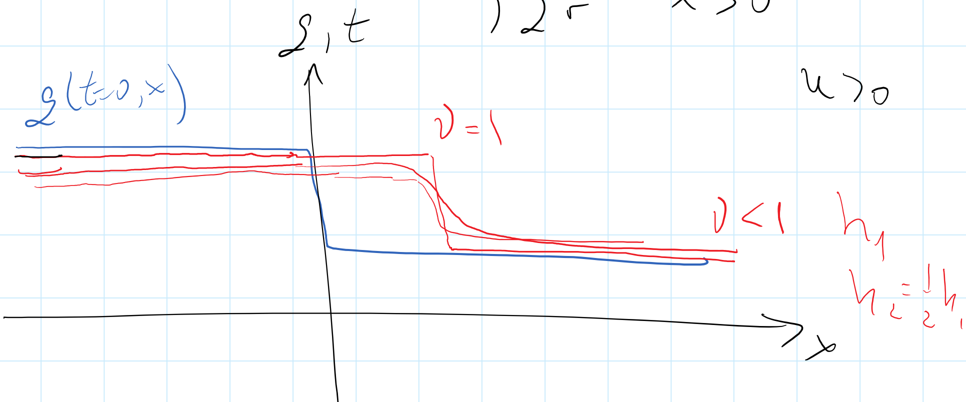



$\mathcal{F}$ (2): $\dfrac{d\hat{\rho}}{dt} = -\xi^2 u\,\hat{\rho}$




IVP $\begin{cases} \rho_t = -u\rho_x \\ \rho(t=0,x) = \begin{cases} \rho_l & x<0 \\ \rho_r & x>0 \end{cases} \end{cases}$

$$\rho(t=0,x) = \begin{cases} \rho_l & \\ \rho_r & x>0 \end{cases}$$



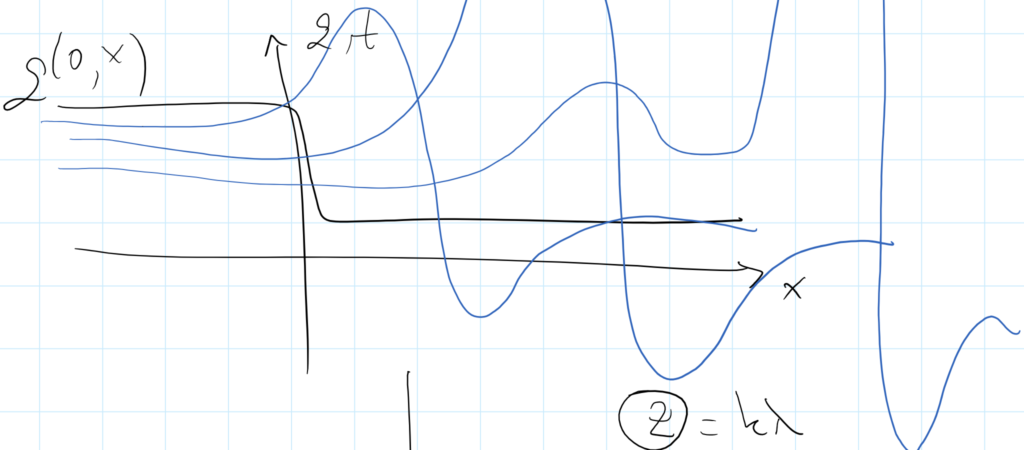

$u>0$

$$\rho_j^{n+1} = \rho_j^n - \nu\left(\rho_j^n - \rho_{j-1}^n\right) \quad \text{up}$$

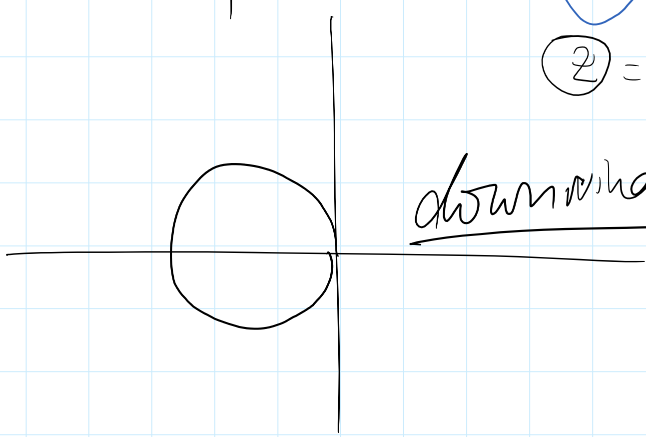
$$\rho_t = -u\rho_x + u h(1-\nu)\rho_{xx}$$

$$\rho_j^{n+1} = \rho_j^n - \nu\left(\rho_{j+1}^n - \rho_j^n\right) \quad \text{down}$$

$$\rho_j^{n+1} = \rho_j^n - \frac{\nu}{2}\left(\rho_{j+1}^n - \rho_{j-1}^n\right) \quad \text{FTCS}$$




$② = k\lambda$



downwind

$$\rho_t = -u\rho_x \qquad \text{advection}$$

$$\frac{d\rho}{dt} = -\frac{u}{2h} A\rho$$

Semidiscretization with centered finite diff

$2h$ , finite diff.

$$A = \begin{pmatrix} 0 & & & \\ -1 & \ddots & 1 & \\ & & \ddots & \\ & & & 0 \end{pmatrix}$$

$A = -A^T$