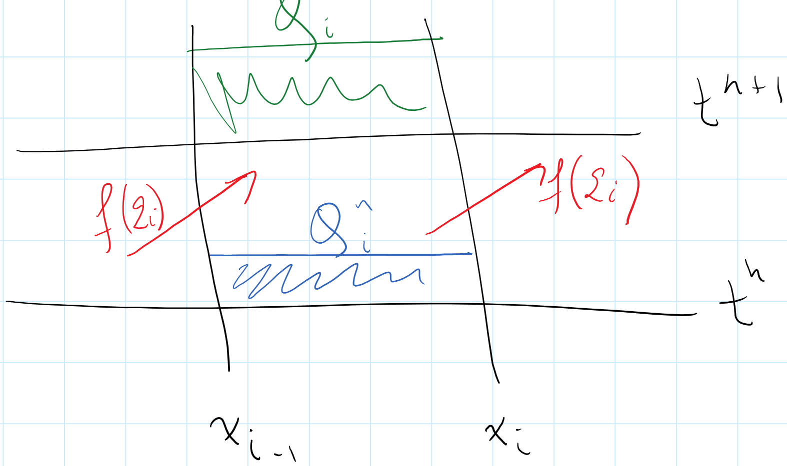
Numerical fluxes

Friday, January 27, 2017 11:23 AM

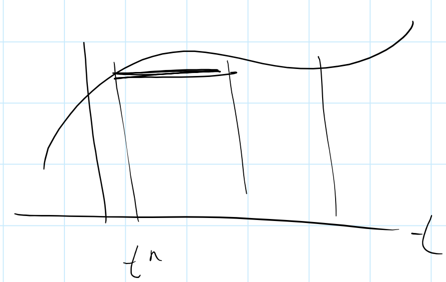
$$q_t + f(q)_x = 0 \xrightarrow{\text{F.V.M.}} Q_i^{n+1} = Q_i^n - \frac{k}{h}\left(F_i^n - F_{i-1}^n\right)$$



(1) $$F_i = \frac{1}{k}\int_{t^n}^{t^{n+1}} f(q(t, x_i))\, dt$$ Numerical flux

Goal: Evaluate numerical flux F based on physical flux $\underline{\underline{f}}$

(2) $$F_i^n = f(Q_i^n)$$



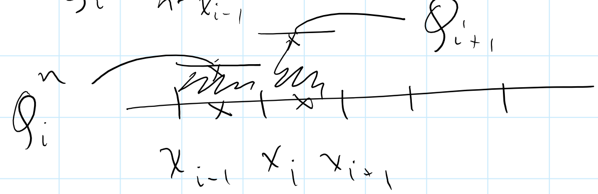
<u>Ex:</u> $$q_t + (uq)_x = 0$$

$$Q_i^{n+1} = Q_i^n - \frac{k}{h}\left(u Q_i^n - u Q_{i-1}^n\right)$$

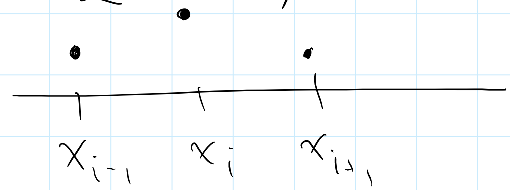
$$= Q_i^n - \nu\left(Q_i^n - Q_{i-1}^n\right)$$ Similar to F.D.M. upwind

FVM	FDM
$Q_i^n \approx \frac{1}{h}\int_{x_{i-1}}^{x_i} q(t^n, x)\, dx$	$Q_i^n \approx q(t^n, x_i)$

$$Q_i^n \simeq \frac{1}{h}\int_{x_{i-1}}^{x_i} q(t^n,x)\,dx$$

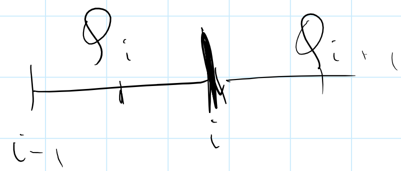


$$Q_i^n \simeq q(t^n, x_i)$$



(F.E.M.: Ciarlet Lions)

(3) $$F_i^n = \frac{1}{2}\left[f(Q_i^n) + f(Q_{i+1}^n)\right]$$



Ex: advection

$$Q_i^{n+1} = Q_i^n - \frac{uk}{2h}\left[Q_{i+1}^n + \cancel{Q_i^n} - \cancel{Q_i^n} - Q_{i-1}^n\right]$$

$$\nu$$

$$Q_i^{n+1} = Q_i^n - \nu\left(Q_{i+1}^n - Q_{i-1}^n\right) \quad \text{FTCS}$$

unstable!

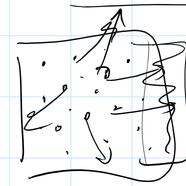
Consider the general problem

$$F_i^n = \mathcal{F}\left(Q_{i-p}^n, Q_{i-p+1}^n, \dots, Q_{i+r}^n\right)$$

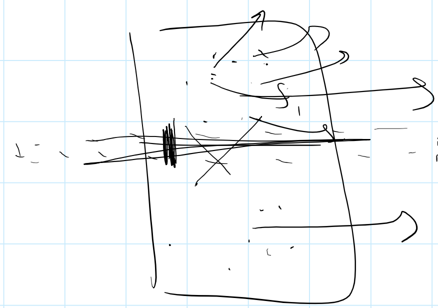
$$F_i^n = \mathcal{F}\left(Q_i^n, Q_{i+1}^n\right) \qquad q_t = \alpha q_{xx}$$

$$F_i^n = \mathcal{F}(\rho_i^n, \rho_{i+1}^n)$$

$$\rho_t = \rho_{xx}$$



$\rho_t = u\rho_x$ at micro level

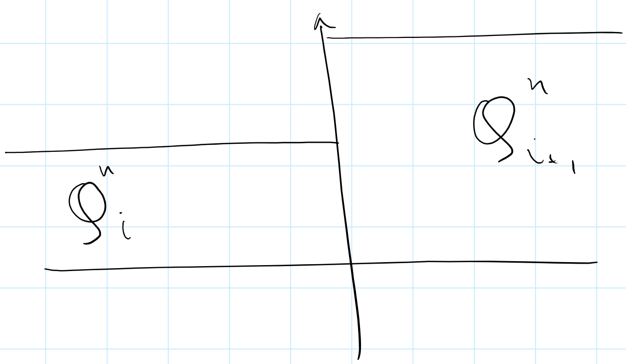


$$\rho u_t + (u\cdot\nabla)u = \nabla p + \nu\nabla^2 u$$

$$\nabla\cdot(\nabla u)$$

Approaches to numerical flux construction

1) Godunov. Consider the Riemann problem at an interface



$$\begin{cases} \rho_t + f(\rho)_x = 0 \\ \rho(t^n, x) = \begin{cases} \rho_i^n & x < x_i \\ \rho_{i+1}^n & x > x_i \end{cases} \end{cases}$$

$\check{\rho}(t, x_i)$ solution

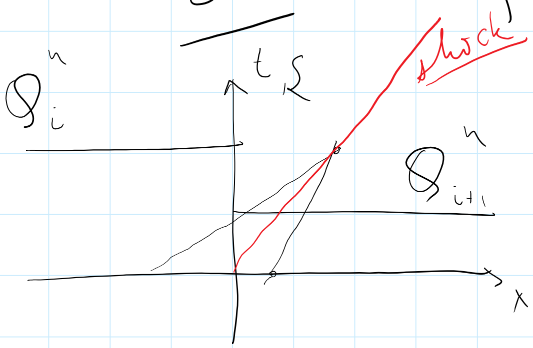
x_i

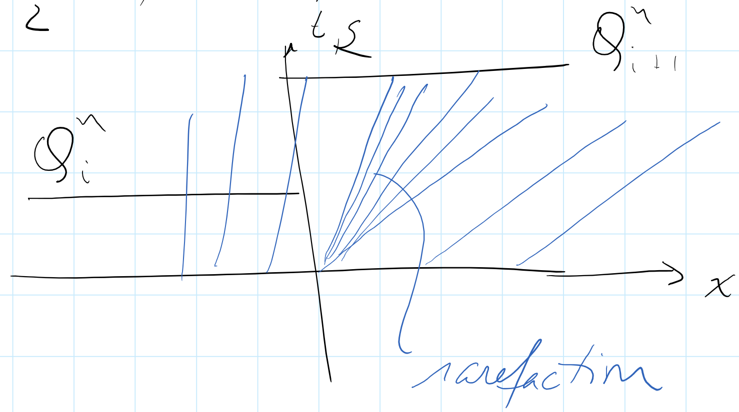
$$F_i^n = \frac{1}{k}\int_{t^n}^{t^{n+1}} \check{\rho}(t, x_i)\, dt$$

Ex 1: $f(\rho) = u\rho$

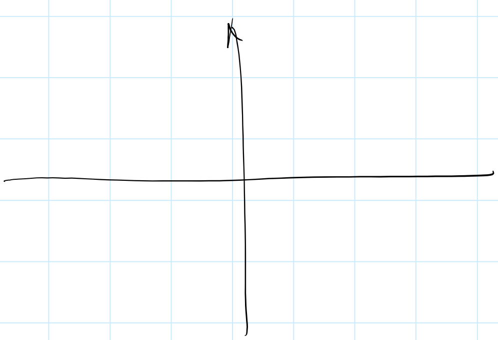
$$\check{\rho}(t, x_i) = \begin{cases} Q_i^n & u > 0 \\ Q_{i+1}^n & u < 0 \end{cases}$$

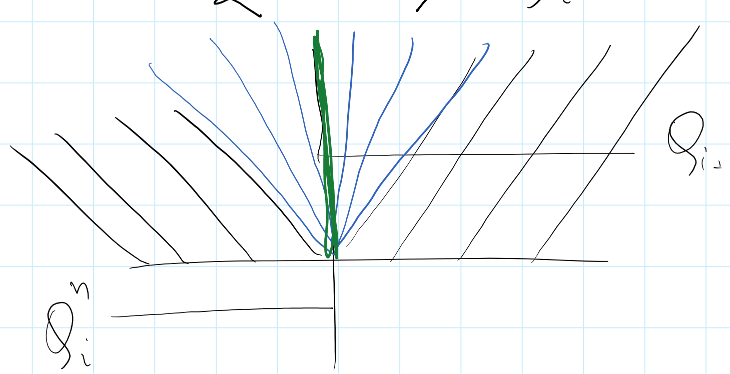
Ex 2: $f(\rho) = \frac{\rho^2}{2}$; $\rho_t + f(\rho)_x = \rho_t + \rho\rho_x = 0$





$$\check{\rho}(t, x_i) = Q_i^n \qquad\qquad \check{\rho}(t, x_i) = Q_i^n$$





$$\check{\rho}(t, x_i) = \rho^0(t)$$

2) Consider $F_i^n = \mathcal{F}(Q_{i-p}, Q_{i-p+1}, \dots, Q_{i+r})$

as an interpolation problem

ENO, WENO

Essentially non-oscillatory: adaptive interpolation stencil

Weighted —— , —— : weighted —"—