Roe solvers

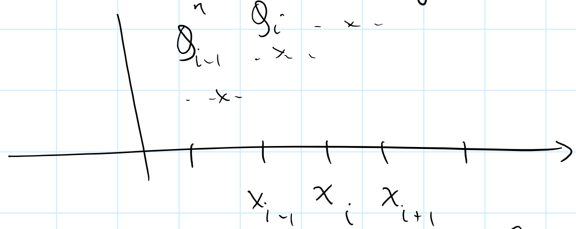
Wednesday, February 8, 2017 11:10 AM

Shallow water equations $q_t + f(q)_x = 0$

$$q = \begin{pmatrix} h \\ hu \end{pmatrix} ; \quad f(q) = \begin{pmatrix} hu \\ hu^2 + \frac{gh^2}{2} \end{pmatrix}$$

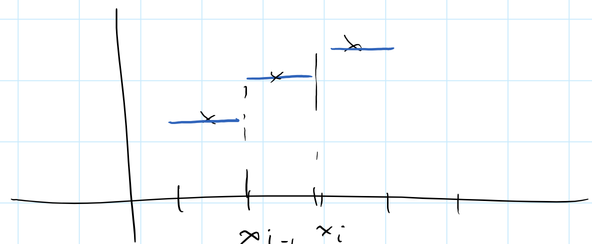
Godunov scheme: Reconstruct Evolve Average algorithm

0) $\{Q_j^n\}$ current state



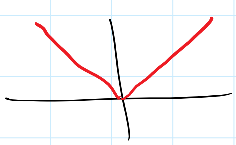
1) Reconstruct $\tilde{q}(t^n, x)$ based upon $\{Q_j^n\}$ (Godunov →)

piecewise constant

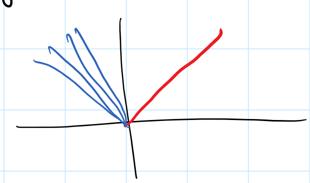


2) Evolve, solving Riemann problem at each interface

Ex: shallow water



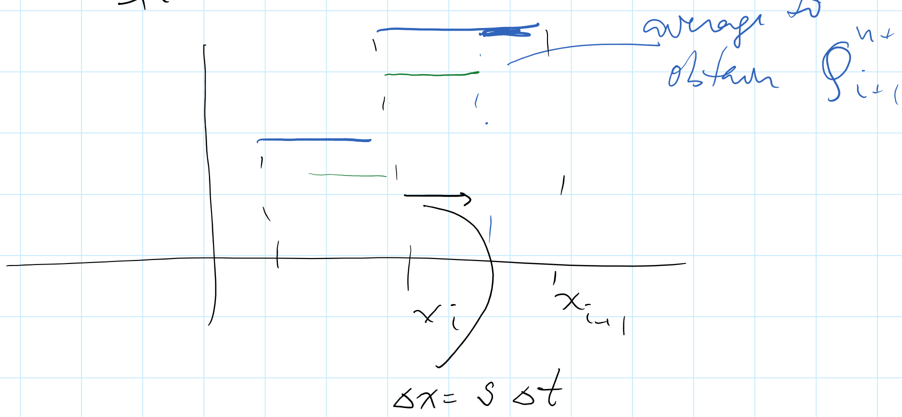
shock - shock



shock-rarefaction

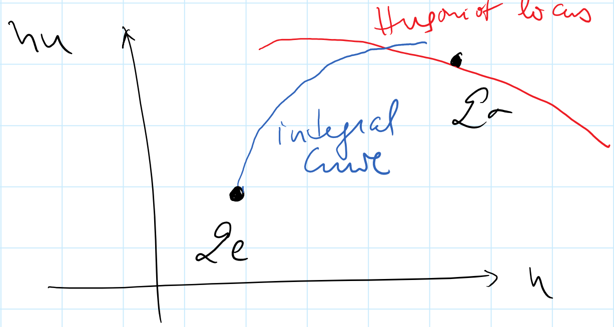
, etc. (impose non-entropy violation)

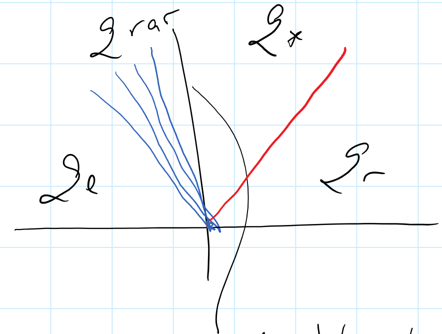
3) Average



Exact Riemann problem solutions exist for very few problems (though important case of gas dynamics is solvable)

Linearized Riemann problems are more economical





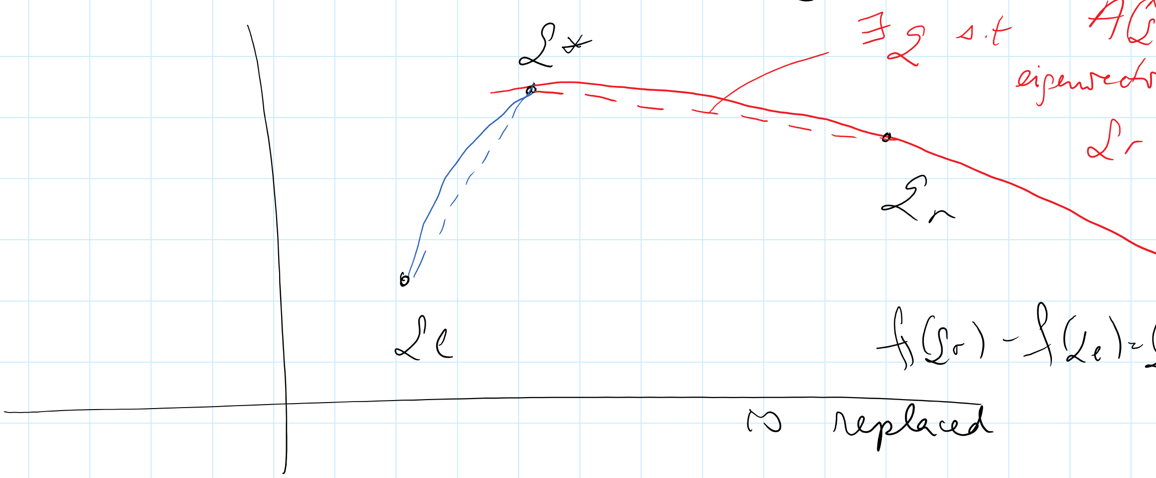
need state at x_i, $t > t^n$ to evaluate flux

<u>Recall</u>: Hugoniot loci result from $s(q_r - q_l) = f(q_r) - f(q_l)$

↳ a discrete formulation of conservation

Integral curves $\dfrac{dq(\xi)}{d\xi} = r(q(\xi))$

$r(q)$ = eigenvector of $\dfrac{\partial f}{\partial q}$.



Roe condition

$f(q_r) - f(q_l) = (q_r - q_l)s$

is replaced

$A(\hat{q})(q_r - q_l) = s(q_r - q_l)$

$\hat{q}$ = Roe average.

Linearized Riemann solver:

1) Computes a particular state ("Roe average" state) s.t. conservation is satisfied as required for accurate shock capturing

2) Solve as in a linear hyperbolic system

Exact, analytical formulations of Roe average are available for fluxes with polynomial dependence of conservative variables, e.g.
- shallow water
- Euler
- MHD