$$q_t + f(q)_x + g(q)_y = 0$$

Dimensional splitting:

1) $q_t + f(q)_x = 0$
$\quad q(t^n, x, y) = Q^n(x,y)$ $\Rightarrow Q^*(x,y)$

2) $q_t + g(q)_y = 0$
$\quad q(t^n, x, y) = Q^*(x,y)$

In general: $\begin{cases} \dfrac{\partial q}{\partial t} = (A+B)q \\ q(0,x) = q_0(x) \end{cases} \Rightarrow$

$$q(t,x) = \underline{e^{(A+B)t}}\, q_0(x) = \underbrace{\underline{e^{At}}\, \underbrace{e^{Bt} q_0(x)}}_{q^*}$$

$$q_t = -f(q)_x - g(q)_y$$

Splitting error computation

$$e^{(A+B)t} = \underline{I} + \underline{\frac{1}{1!}(A+B)t} + \frac{1}{2!}(A+B)^2 t^2 + \ldots$$

$$e^{At} e^{Bt} = \left(I + \frac{1}{1!}At + \frac{1}{2!}A^2t^2 + \ldots\right)\left(I + \frac{1}{1!}Bt + \frac{1}{2!}B^2t^2 \ldots\right)$$

$$= \underline{I} + \underline{\frac{1}{1!}(A+B)t} + \frac{1}{2!}(A^2+B^2)t^2 +$$

$$+ ABt^2 + \ldots$$

$$(A+B)^2 = (A+B)(A+B) = A^2 + BA + AB + B^2$$

if $[A,B] = AB - BA \neq 0 \Rightarrow$ splitting error $\Rightarrow$ scheme 1^{st}-order accurate.

Strang splitting

$$e^{(A+B)t} \simeq e^{At} e^{Bt} \quad \text{(1st-order accurate)}$$

$$e^{(A+B)t} \simeq e^{\frac{1}{2}At} e^{Bt} e^{\frac{1}{2}At} \quad \text{(2nd-order accurate)}$$

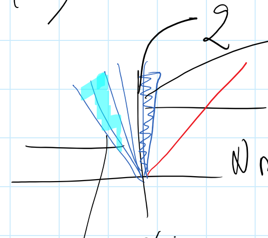
Godunov flux

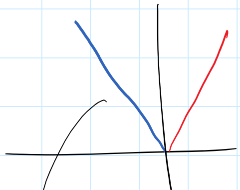
$$Q_i^{n+1} = Q_i^n - \frac{k}{h}\left(F_{i+1}^n - F_i^n\right)$$

k = time step ; h = space step

$$F_i^n = \frac{1}{k}\int_{t^n}^{t^{n+1}} f(q(t,x_i))\,dt$$

$$= \frac{1}{h}\int_{t^n}^{t^{n+1}} f(\hat{q}(x_i))\,dt = f(\hat{q}(x_i))$$





→ The linearized rarefaction jump has to be distributed between right-going & left-going waves

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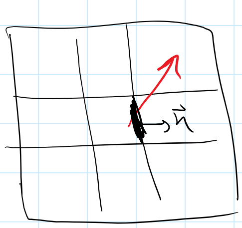
Transverse Riemann problem



In 2D part of the fluctuation associated with normal direction Riemann problem has to be distributed to cells one row above/below

(a.k.a. cross-terms)

Neglect cross-terms : consistent approx., but with more stringent stability restriction explicit: 2D $CFL \leq 0.5$

3D $CFL \leq 0.33$