Overview

- Dissipative systems

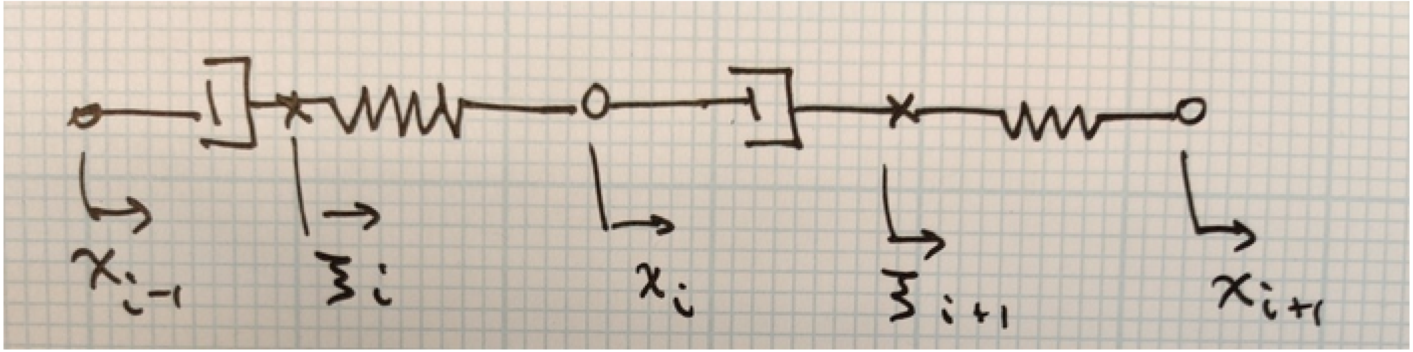

Figure 1. Maxwell elements in series

- Linear array of Maxwell elements

$$
\begin{aligned}
0 &= k(x_i - \xi_i) - c(\dot{\xi}_i - \dot{x}_{i-1}) \\
m\,\ddot{x}_i &= c(\dot{\xi}_{i+1} - \dot{x}_i) - k(x_i - \xi_i) \\
0 &= k(x_{i+1} - \xi_{i+1}) - c(\dot{\xi}_{i+1} - \dot{x}_i)
\end{aligned}
$$

- Laplace transform

$$
\begin{aligned}
0 &= k(X_i - \Xi_i) - s\,c(\Xi_i - X_{i-1}) - c\,x_{i-1}(0) \\
s^2 m X_i - m\,x_i(0) &= s\,c(\Xi_{i+1} - X_i) - k(X_i - \Xi_i) + c\,x_i(0)
\end{aligned}
$$

$$0 = k(X_{i+1} - \Xi_{i+1}) - sc(\Xi_{i+1} - X_i) - cx_i(0)$$

Viscoelastic continuum limit

- Eliminate internal coordinates Ξ_i, Ξ_{i+1}

$$\Xi_i = \frac{1}{k+sc}(kX_i + scX_{i-1}) - \frac{c}{k+sc}x_{i-1}(0)$$

- Replace internal coordinates in momentum balance for x_i

$$\begin{aligned} s^2 m X_i &= \frac{sck}{k+sc}X_{i+1} - \frac{2sck}{k+sc}X_i + \frac{sck}{k+sc}X_{i-1} + \\ &\quad \frac{mk + (sm+k)c}{k+sc}x_i(0) - \frac{kc}{k+sc}x_{i-1}(0) \end{aligned}$$

- Compare to harmonic oscillator

$$s^2 m X_i = k(X_{i+1} - 2X_i + X_{i-1}) + m x_i(0)$$

- Verify against non-dissipative result in $c \to \infty$ limit, $x_{i-1}(0) \to x_i(0)$

$$s^2 m X_i = kX_{i-1} - 2kX_i + kX_{i+1} + m x_i(0)$$

- Fourier transform in space

$$U_{tt}(t,k) = -k^2 U(t,k) \Rightarrow U_{tt} + k^2 U = 0 \Rightarrow U(t,k) = A\cos(kt) + B\sin(kt)$$

- Array of nonlinear oscillators

$$\begin{aligned} \ddot{x}_i &= x_{i+1} - 2x_i + x_{i-1} + \alpha[(x_{i+1} - x_i)^2 - (x_i - x_{i-1})^2] \\ \ddot{x}_i &= x_{i+1} - 2x_i + x_{i-1} + \beta[(x_{i+1} - x_i)^3 - (x_i - x_{i-1})^3] \end{aligned}$$

Continuum limit of FUP problems

- Quadratic nonlinearity

$$\ddot{x}_i = x_{i+1} - 2x_i + x_{i-1} + \alpha[(x_{i+1} - x_i)^2 - (x_i - x_{i-1})^2] \to u_{tt} = u_{xx} + \nu u_x u_{xx}$$

- Fourier transform in space

$$U_{tt}(t,k) = -k^2 U(t,k) - i\nu k^3 \int U(l)U(k-l)\,\mathrm{d}l$$

$$U_{tt}(t,k) = -k^2 U(t,k) - i\nu k^3 \sum_l U(l)U(k-l)$$