Overview

- Microscopic systems are exact but impractical
- Reduce number of degrees of freedom (DOFs)
- Extract a constitutive law from microscopic

- Consider a linear elastic-dissipative model, $\boldsymbol{x} \in \mathbb{R}^m$, $m \gg 1$

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{D}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f}$$

- Choose an orthonormal basis $\boldsymbol{B} \in \mathbb{R}^{m \times n}$, $n \ll m$, $\boldsymbol{x} = \boldsymbol{B}\boldsymbol{y}$, $\boldsymbol{P} = \boldsymbol{B}\boldsymbol{B}^T$

$$\boldsymbol{B}^T\boldsymbol{M}\boldsymbol{B}\ddot{\boldsymbol{y}} + \boldsymbol{B}^T\boldsymbol{D}\,\boldsymbol{B}\,\dot{\boldsymbol{y}} + \boldsymbol{B}^T\boldsymbol{K}\boldsymbol{B}\,\boldsymbol{y} = \boldsymbol{B}^T\boldsymbol{f} \Rightarrow$$

- Obtain reduced system

$$\boldsymbol{M}_B\ddot{\boldsymbol{y}} + \boldsymbol{D}_B\dot{\boldsymbol{y}} + \boldsymbol{K}_B\boldsymbol{y} = \boldsymbol{f}_B$$

- Suppose system was a string with fixed ends, Sturm-Liuville problem has eigenbasis $\{\sin x, \sin 2x, \ldots\}$

$$\boldsymbol{B} = [\ \sin x \quad \sin 2x \quad \ldots \quad \sin nx\ ]$$

- Seek correlations of observed data $\boldsymbol{x}_j = \boldsymbol{x}(t_j)$, $\langle \boldsymbol{x}_j \rangle = \boldsymbol{0}$ (deviations from equil)

$$\boldsymbol{X} = [\ \boldsymbol{x}_1 \quad \ldots \quad \boldsymbol{x}_n\ ] = \boldsymbol{Q}\boldsymbol{\Sigma}\boldsymbol{R}^T, \boldsymbol{C} = \boldsymbol{X}\boldsymbol{X}^T = \boldsymbol{Q}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^T\boldsymbol{Q}^T$$

- $\boldsymbol{C}$ is s.p.d., hence unitarily diagonalizable $\boldsymbol{C} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^T$
- $\boldsymbol{X} = \sum_{k=1}^{\min(m,n)} \sigma_k \boldsymbol{q}_k \boldsymbol{q}_k^T \cong \sum_{k=1}^{K} \sigma_k \boldsymbol{q}_k \boldsymbol{q}_k^T$, K cutoff

- $\boldsymbol{Q}_k = [\ \boldsymbol{q}_1 \ \ \dots \ \ \boldsymbol{q}_k\ ]$, s step size in reduction of DOFs, $\boldsymbol{x} \in \mathbb{R}^m$, $\boldsymbol{y}_1 \in \mathbb{R}^{m-s}$

$$\boldsymbol{x} = \boldsymbol{Q}_{m-s}\boldsymbol{y}_1 = \boldsymbol{Q}_{m-s}\boldsymbol{Q}_{m-2s}\boldsymbol{y}_{m-2} = \boldsymbol{Q}_{m-s}\dots\boldsymbol{Q}_{m-ks}\,\boldsymbol{y}_{m-k}$$

- Extend to nonlinear model reduction (DNN)

$$\boldsymbol{x} = \boldsymbol{Q}_{m-s} \circ \boldsymbol{\sigma} \circ \dots \circ \boldsymbol{Q}_{m-(k-1)s} \circ \boldsymbol{\sigma} \circ \boldsymbol{Q}_{m-ks}\,\boldsymbol{y}_{m-k}$$