Definition. $(\mathbb{X}, \mathcal{U})$ *is a topological space if* $\mathcal{U}$ *is a collection of open subsets of* $\mathbb{X}$ *and:*

i. $\mathbb{X} \in \mathcal{U}$, $\varnothing \in \mathcal{U}$;

ii. $U_1, U_2 \in \mathcal{U} \Rightarrow U_1 \cap U_2 \in \mathcal{U}$;

iii. $U_i \in \mathcal{U}$, $i \in I$, $\bigcup_{i \in I} U_i \in \mathcal{U}$.

Definition. $\mathcal{B}$, *a collection of subsets of* $\mathbb{X}$, *is a basis for a topology on point set* $\mathbb{X}$ *if:*

i. $\forall x \in \mathbb{X}$, $\exists B \in \mathcal{B}$ *such that* $x \in B$;

ii. $x \in B_1 \cap B_2$, $B_1, B_2 \in \mathcal{B} \Rightarrow \exists B_3 \in \mathcal{B}$ *such that* $x \in B_3 \subseteq B_1 \cap B_2$

The topology $\mathcal{U}$ generated by $\mathcal{B}$ is $\{U : U \subseteq \mathbb{X} \text{ and } x \in U \Rightarrow \exists B \in \mathcal{B} \text{ such that } x \in B \subseteq U\}$. The topology is constructed explicitly by all unions of finite intersections $\bigcap_{i=1}^{n} B_i$.

Example 1. $\mathbb{X} = \mathbb{R}$, $\mathcal{B} = \{(a, b), a < b, a, b \in \mathbb{R}\}$, standard real line topology.

Definition. *A complex is a decomposition of a topological space into topologically simple pieces.*

Definition. $u_0, \ldots, u_k \in \mathbb{R}^d$, $x = \sum_{i=0}^{k} \lambda_i u_i$ *is an affine combination if* $\sum_{i=0}^{k} \lambda_i = 1$

Definition. *The affine hull of* $u_0, \ldots, u_k \in \mathbb{R}^d$ *is the set of all affine combinations.*

Definition. $u_0, \ldots, u_k \in \mathbb{R}^d$ *are affinely independent if* $\sum_{i=0}^{k} \lambda_i u_i = \sum_{i=0}^{k} \mu_i u_i \Leftrightarrow \lambda_i = \mu_i$.

Definition. *The affine hull of affinely independent* $u_0, \ldots, u_k \in \mathbb{R}^d$ *is a* k*-plane.*

Proposition. $u_0, \ldots, u_k \in \mathbb{R}^d$ *are affinely independent iff vectors* $u_i - u_0$, $i = 1, \ldots, k$ *are linearly independent.*

Definition. $u_0, \ldots, u_k \in \mathbb{R}^d$, $x = \sum_{i=0}^{k} \lambda_i u_i$ *is a convex combination if* $\sum_{i=0}^{k} \lambda_i = 1$ *and* $\lambda_i \geqslant 0$.

Definition. *The convex hull of* $u_0, \ldots, u_k \in \mathbb{R}^d$ *is the set of all convex combinations.*

Definition. *The convex hull of affinely independent* $u_0, \ldots, u_k \in \mathbb{R}^d$ *is a* k*-simplex.*

Notation. $\sigma = \mathrm{conv}\{u_0, \ldots, u_k\}$ *is the convex hull of* $u_0, \ldots, u_k \in \mathbb{R}^d$.

$\dim \sigma = k$ if $u_0, \ldots, u_k$ are affinely independent.

Definition. $\tau = \mathrm{conv}\{u_{i_0}, \ldots, u_{i_j}\}$ *is a face of* $\sigma = \mathrm{conv}\{u_0, \ldots, u_k\}$ *if* $j \geqslant 0$, $0 \leqslant i_0, \ldots, i_j \leqslant k$, *i.e., the convex hull of a non-empty subset of points, denoted as* $\tau \leqslant \sigma$. *If* $j < k$, *it is a proper face, denoted as* $\tau < \sigma$.

Definition. *The boundary of a convex hull* σ *is the union of all proper faces, denoted as* $\mathrm{bd}\,\sigma$. *The complement of the boundary of a convex hull is the interior of the convex hull,* $\mathrm{int}\,\sigma = \sigma - \mathrm{bd}\,\sigma$.

$x = \sum_{i=0}^{k} \lambda_i u_i \in \mathrm{int}\,\sigma$ if all $\lambda_i > 0$. $\forall x \in \sigma$ belongs to interior of exactly one face.

Definition. *A simplicial complex is a finite collection of simplices* $K = \{\sigma_i\}$ *such that:*

i. $\sigma \in K$ *and* $\tau \leqslant \sigma \Rightarrow \tau \in K$, *and*

ii. $\sigma, \sigma_0 \in K \Rightarrow \mathrm{either}\ \sigma \cap \sigma_0 = \varnothing$ *or a face of both* $\sigma \cap \sigma_0 \leqslant \sigma$ *and* $\sigma \cap \sigma_0 \leqslant \sigma_0$.

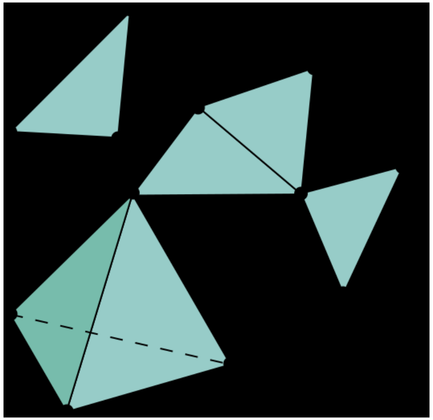
Dimension of a simplicial complex $\dim K = \max_i \dim \sigma_i$

Definition. *The underlying space of a simplicial complex, $|K| = \{\sigma_i\}$ is $\bigcup_i \sigma_i$.*

Definition. *A triangulation of topological space $(\mathbb{X}, \mathcal{U})$ is a simplicial complex K together with a homeomorphism between $\mathbb{X}$ and $|K|$.*

Definition. *A subcomplex of K is a simplicial complex $L \subseteq K$.*

Definition. *The j-skeleton $K^{(j)}$ of K is the subcomplex consisting of all simplices of dimension at most j.*



The 0-skeleton, $K^{(0)}$ is the vertex set.