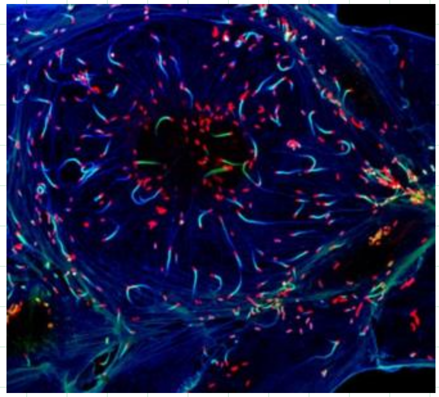
Topological simplification

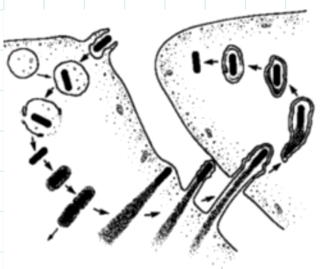
Friday, February 10, 2017 12:26 PM

1) Background

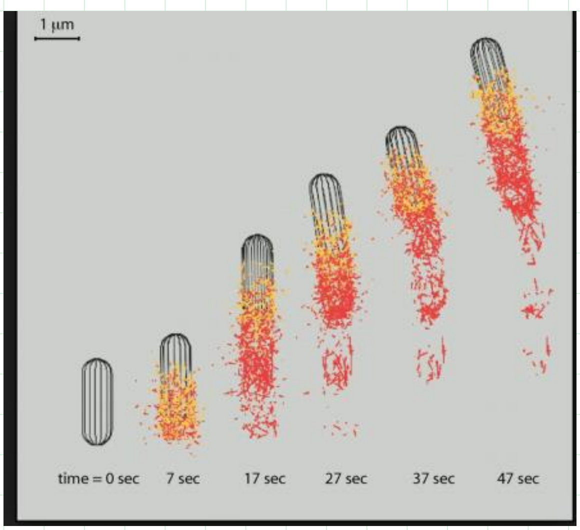
- L. mono is an infectious bacterium
- L. mono moves by redirecting cell cytoskeleton formation



L. mono infestation.
Bacterium stained in red
Trajectory visualised in various colors



Infectious cycle



Numerical simulation
PLOS Biology 2004 2(12): e412
J. Alberts & G. Odell

Results of numerical simulation

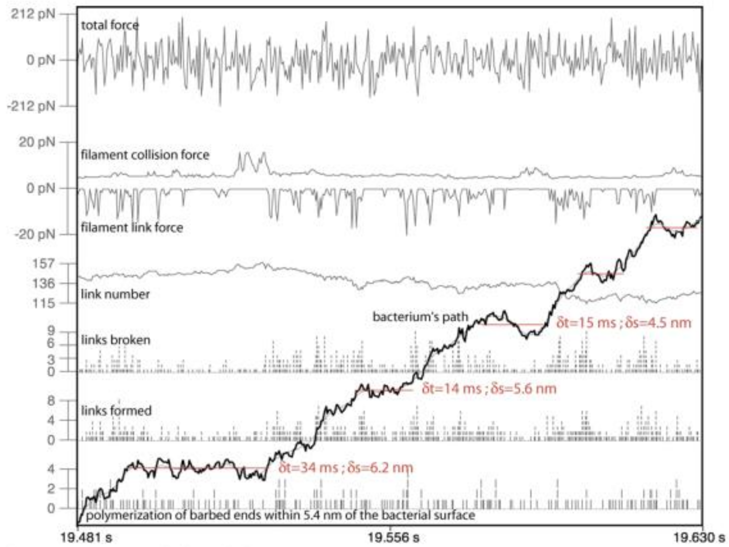


Figure 5. System Outcome Profiles for Several Adjacent Pauses

Pauses are shown by red horizontal lines in a single simulation run in which the Brownian force multiplier was 1 (unconstrained bacterium). Listed for each pause are the pause duration (δt) and distance to the following pause (δs) as reported by our line-fitted analysis. The vertical line segments in the lower half of the figure show discrete events. From the bottom, these are the number of polymerization events on filaments very close to the bacterium, the number of new links formed between the bacterium and filaments, and the number of these bacterium–filament links broken. Above these are plotted the total number of bacterium–filament links. At the top, the net filament link force and the net filament collision force are given in picoNewtons (see Figure 1B), along with the total force. Some general trends aligned with pauses are apparent, such as decreased actin and link dynamics during a pause, but any characteristic biochemical or force response is obscured by the Brownian agitation of the bacterium.

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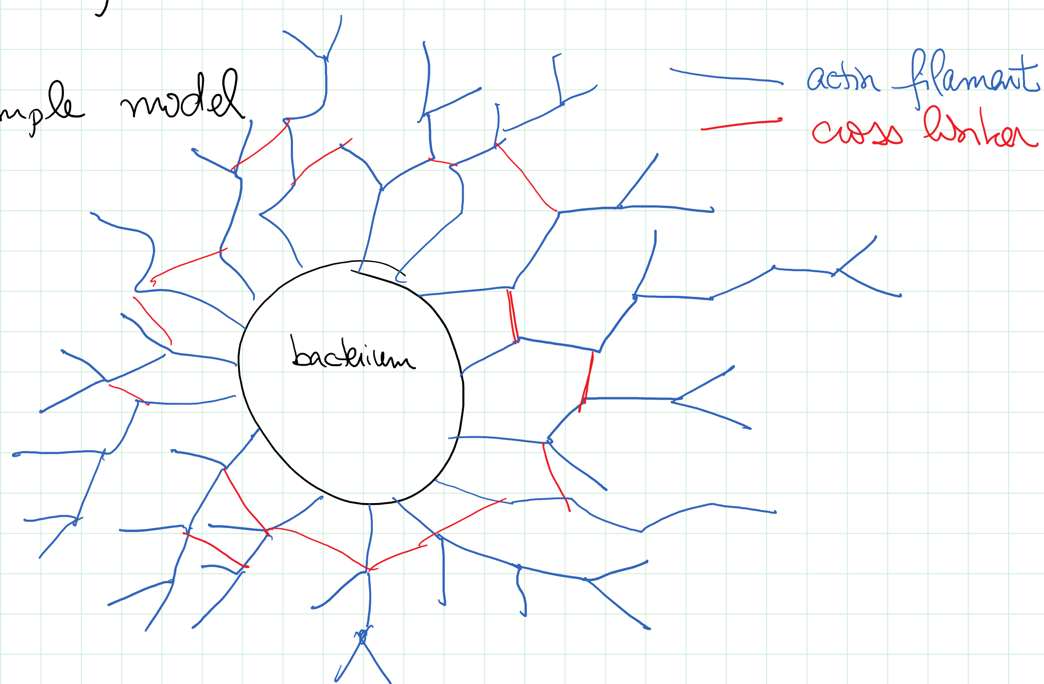
Mathematical research questions:

1) What is the correct stochastic model of the resulting behavior?

2) How to characterise actin network rupture

Simple model

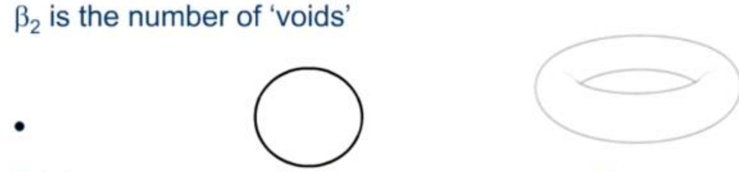


Topological simplification problem: Characterize "rupture", i.e., breaking enough links in the network to release stored elastic energy.

Approach: Edelsbrunner, Letscher, Zomorodian.
Use topological complexity of point sets in $\mathbb{R}^3$.

- General set in $\mathbb{R}^3$: $(\beta_0, \beta_1, \beta_2)$ = Betti number set

  – β_0 is the number of connected components
  – β_1 is the number of tunnels or handles
  – β_2 is the number of 'voids'



Point	Circle	Torus
$\beta_0 = 1$	$\beta_0 = 1$	$\beta_0 = 1$
$\beta_1 = 0$	$\beta_1 = 1$	$\beta_1 = 2$
$\beta_2 = 0$	$\beta_2 = 0$	$\beta_2 = 1$

- Change l length at which cross links are formed
  $l = 0 \Rightarrow \beta_0 =$ n. of filaments
  $l = l_{critical} \Rightarrow \beta_0 = 1$
  $l < l_{critical} \Rightarrow$ rupture