SciComp Practice Exam 5/12/14

*Answer the following questions explaining all steps that lead to a solution. Results presented without moti-vation will **not** receive any credit.*

1. Approximate $I = \int_0^1 \log(x) \sin(x^2/100)\, dx$ to a relative error $\varepsilon = 10^{-4}$.

2. Assume the real-valued sequence $\{x_n\}_{n\in\mathbb{N}}$ exhibits first-order convergence to $a \in \mathbb{R}$.

   a) What is the limit of the sequence

   $$y_n = \frac{x_n\, x_{n+2} - x_{n+1}^2}{x_{n+2} - 2x_{n+1} + x_n} \; ?$$

   b) At what rate does $\{y_n\}_{n\in\mathbb{N}}$ converge?

3. Find the best approximation of $\sqrt{x}$ by a first-degree polynomial on the interval $[0,1]$.

4. Approximate $x(1/2)$ to relative error $\varepsilon = 10^{-3}$ with $x(t)$ solution of the two-point boundary value problem

   $$\begin{aligned} x'' + 2x' + 10x &= 0 \\ x(0) = 1,\, x(1) &= 2 \end{aligned} \;.$$

5. Consider the Cholesky factorization $A = LL^T$ of $A \in \mathbb{R}^{m\times m}$, symmetric positive definite. Given L write the pseudocode to compute A with minimal use of computational resources (memory, floating point operations). Specify the algorithm complexity.

6. Determine the condition number of the algorithm

$$x \to \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$