SciComp Practice Exam 5/19/14

Answer the following questions explaining all steps that lead to a solution. Results presented without motivation will ***not*** *receive any credit.*

1. Construct a fourth-order accurate approximation formula of $f'(x_0)$, based on values of $f\colon \mathbb{R} \to \mathbb{R}$ at points $x_i = x_0 + ih$, $i \in \mathbb{Z}$. Provide an estimate of the step size h that provides smallest relative error of the approximation in double precision floating point computation. Assume $f \in C^\infty(\mathbb{R})$.

2. Write an algorithm to evaluate $f(x) = (1 - \cos x)/x$ with minimal loss of precision in floating point arithmetic.

3. Let $p \in (0, \infty)$. What is the value of

$$x = \sqrt{p + \sqrt{p + \ldots + \sqrt{p + \ldots}}}\,?$$

4. Show that $\|\|\colon \mathbb{R}^{m\times m} \to \mathbb{R}_+$ defined as

$$\|A\| = \sum_{i=1}^{m} \sum_{j=1}^{m} |a_{ij}|,$$

is a matrix norm, and not subordinate to any vector norm.

5. For given $A \in \mathbb{R}^{m\times n}$, $\alpha \in \mathbb{R}_+$, define $F\colon \mathbb{R}^n \to \mathbb{R}_+$ by

$$F(x) = \|Ax - b\|_2^2 + \alpha \|x\|_2^2.$$

Prove that solving

$$\min_x F(x),$$

is equivalent to solving

$$(A^T A + \alpha I)\, x = A^T b. \tag{1}$$

For x solution of (1) compute $F(x + h)$ $(h \in \mathbb{R}_+)$ in terms of $F(x), \alpha, h, A$.

6. Determine the end conditions for a cubic spline interpolation $S(x)$ that minimize

$$\int_a^b [S''(x)]^2 \, \mathrm{d}x.$$