

Scientific Computation Comprehensive Examination Practice Questions

Answer the following questions explaining all steps that lead to a solution. Partial credit will be awarded for presenting a viable solution strategy. No credit will be given to computations presented without motivation. Your goal is to present skill in formulating precise mathematical statements, and demonstrate understanding of theoretical material.

1. State the conditions on $g(x)$ that ensure third-order convergence to the root r of the equation $f(x) = 0$ for the sequence defined by $x_{n+1} = F(x_n)$, $F(x) = x + f(x)g(x)$.
2. State an algorithm that requires $\mathcal{O}(n^2)$ operations to compute the QR decomposition of $A = R + uv^T$, with R an upper triangular matrix.
- 3.

- a) Determine the order of convergence of the sequence

$$S_n = \frac{1}{2} \sum_{j=0}^{n-1} (f_j + f_{j+1})(g_{j+1} - g_j), \quad (1)$$

to the integral

$$I(f, g) = \int_0^1 f(x) g'(x) dx,$$

with $h = 1/n$, $n \in \mathbb{N}$, $f_j \equiv f(jh)$, $g_j \equiv g(jh)$.

- b) Use quadrature rule (1) to provide approximation S_2 of

$$J = \int_0^1 \frac{e^x}{\sqrt{1-x^2}} dx,$$

and estimate the error $e_2 = |S_2 - J|$.